Name ________________________________ **Date** ______________ **Class** ____________

Section 1-1 Functions

Goal: To evaluate function values and to determine the domain of functions

The domain of the following functions will be the set of real numbers unless it meets one of the following conditions:
1. The function contains a fraction whose denominator has a variable.
   The domain of such a function is the set of real numbers EXCEPT the values of the variable that make the denominator zero.
2. The function contains an even root (square root $\sqrt{\ }$, fourth root $\sqrt[4]{\ }$, etc.).
   The domain of such a function is limited to values of the variable that make the radicand (the part under the radical) greater than or equal to 0.

1. Evaluate the following function at the specified values of the independent variable and simplify the results.

$f(x) = 4x - 5$

a) $f(1) = 4(1) - 5$
$f(1) = 4 - 5$
$f(1) = -1$

b) $f(-3) = 4(-3) - 5$
$f(-3) = -12 - 5$
$f(-3) = -17$

c) $f(x-1) = 4(x-1) - 5$
$f(x-1) = 4x - 4 - 5$
$f(x-1) = 4x - 9$

d) $f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 5$
$f\left(\frac{1}{4}\right) = 1 - 5$
$f\left(\frac{1}{4}\right) = -4$

In problems 2–10 evaluate the given function for $f(x) = x^2 + 1$ and $g(x) = x - 4$.

2. $(f+g)(3) = f(3) + g(3)$
$= 10 + (-1)$
$(f+g)(3) = 9$

$f(3) = (3)^2 + 1$
$f(3) = 9 + 1$
$f(3) = 10$

$g(3) = 3 - 4$
$g(3) = -1$

3. $(f-g)(2c) = f(2c) - g(2c)$ $\quad f(2c) = (2c)^2 + 1$ $\quad g(2c) = 2c - 4$

$= 4c^2 + 1 - (2c - 4)$ $\quad f(2c) = 4c^2 + 1$

$(f-g)(2c) = 4c^2 - 2c + 5$

4. $(fg)(-4) = f(-4)g(-4)$ $\quad f(-4) = (-4)^2 + 1$ $\quad g(-4) = -4 - 4$

$= (17)(-8)$ $\quad f(-4) = 16 + 1$ $\quad g(-4) = -8$

$(fg)(-4) = -136$ $\quad f(-4) = 17$

5. $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)}$ $\quad f(0) = (0)^2 + 1$ $\quad g(0) = 0 - 4$

$= \frac{1}{-4}$ $\quad f(0) = 0 + 1$ $\quad g(0) = -4$

$\left(\frac{f}{g}\right)(0) = -\frac{1}{4}$ $\quad f(0) = 1$

6. $2 \cdot g(-2) = 2(-6)$ $\quad g(-2) = -2 - 4$

$2 \cdot g(-2) = -12$ $\quad g(-2) = -6$

7. $3 \cdot f(4) - 2 \cdot g(-1) = 3(17) - 2(-5)$ $\quad f(4) = (4)^2 + 1$ $\quad g(-1) = -1 - 4$

$= 51 + 10$ $\quad f(4) = 16 + 1$ $\quad g(-1) = -5$

$3 \cdot f(4) - 2 \cdot g(-1) = 61$ $\quad f(4) = 17$

8. $\frac{f(3) - g(2)}{f(1)} = \frac{10 - (-2)}{2}$ $\quad f(3) = (3)^2 + 1$ $\quad f(1) = (1)^2 + 1$ $\quad g(2) = 2 - 4$

$= \frac{12}{2}$ $\quad f(3) = 9 + 1$ $\quad f(1) = 1 + 1$ $\quad g(2) = -2$

$\frac{f(3) - g(2)}{f(1)} = 6$ $\quad f(3) = 10$ $\quad f(1) = 2$

9. $$\frac{g(-1+h)-g(-1)}{h}=\frac{h-5-(-5)}{h}$$
$$=\frac{h}{h}$$
$$\frac{g(-1+h)-g(-1)}{h}=1$$

$g(-1+h)=-1+h-4$
$g(-1+h)=h-5$

$g(-1)=-1-4$
$g(-1)=-5$

10. $$\frac{f(2+h)-f(2)}{h}=\frac{h^2+4h+5-(5)}{h}$$
$$=\frac{h^2+4h}{h}$$
$$\frac{f(2+h)-f(2)}{h}=h+4$$

$f(2+h)=(2+h)^2+1$
$f(2+h)=h^2+4h+4+1$
$f(2+h)=h^2+4h+5$

$f(2)=(2)^2+1$
$f(2)=4+1$
$f(2)=5$

In problems 11–18 find the domain of each function.

11. $g(x)=\dfrac{5}{x-2}$

The domain is restricted by the denominator. Since it cannot equal zero, the domain is all real numbers except 2.

12. $f(x)=\dfrac{2x}{3x+7}$

The domain is restricted by the denominator. Since it cannot equal zero, the domain is all real numbers except $-\dfrac{7}{3}$.

13. $h(t)=\sqrt[4]{1-2t}$

The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $t\le\dfrac{1}{2}$.

14. $g(x)=1-2x^2$

There are no restrictions on the domain, therefore the domain is all real numbers.

15. $f(x) = \sqrt[3]{x+4}$

There are no restrictions on the domain since it has an odd root, therefore the domain is all real numbers.

16. $h(w) = \sqrt{w-3}$

The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $w \geq 3$.

17. $f(x) = 2x^3 + 5x^2 - x + 17$

There are no restrictions on the domain, therefore the domain is all real numbers.

18. $g(x) = \frac{2x^3}{5}$

There are no restrictions on the domain since there is no variable in the denominator, therefore the domain is all real numbers.

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Section 1-2 Elementary Function: Graphs and Transformations

Goal: To describe the shapes of graphs based on vertical and horizontal shifts and reflections, stretches, and shrinks

Basic Elementary Functions:

$f(x) = x$	Identity function
$h(x) = x^2$	Square function
$m(x) = x^3$	Cube function
$n(x) = \sqrt{x}$	Square root function
$p(x) = \sqrt[3]{x}$	Cube root function
$g(x) = \lvert x \rvert$	Absolute value function

In problems 1–14 describe how the graph of each function is related to the graph of one of the six basic functions. State the domain of each function. (Do not use a graphing calculator and do not make a chart.)

1. $g(x) = x^2 - 4$

The graph is the square function that is shifted down 4 units. There are no restrictions on the domain, therefore, the domain is all real numbers.

2. $f(x) = \sqrt{x} + 5$

The graph is the square root function that is shifted up 5 units. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 0$.

3. $f(x) = -\sqrt{x}$

The graph is the square root function that is reflected over the x–axis. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 0$.

4. $f(x) = \sqrt[3]{x-2}$

The graph is the cube root function that is shifted 2 units to the right. There are no restrictions on the domain since it has an odd root, therefore the domain is all real number.

5. $g(x) = (x-5)^2 - 3$

The graph is the square function that is shifted to the right 5 units and down 3 units. There are no restrictions on the domain, therefore, the domain is all real numbers.

6. $f(x) = -x^2 + 1$

The graph is the square function that is reflected over the x–axis and shifted up 1 unit. There are no restrictions on the domain, therefore, the domain is all real numbers.

7. $g(x) = 2 - \sqrt{x-4}$

The graph is the square root function that is shifted 4 units to the right, reflected over the x–axis, and shifted 2 units up. The domain is restricted by the radicand since it has an even root. Since the radicand must be greater than or equal to zero, the domain is $x \geq 4$.

8. $h(x) = |x+5|$

The graph is the absolute value function that is shifted 5 units to the left. There are no restrictions on the domain, therefore, the domain is all real numbers.

9. $g(x) = \sqrt[3]{x} - 3$

The graph is the cube root function that is shifted 3 units down. There are no restrictions on the domain since it has an odd root, therefore the domain is all real number.

10. $f(x) = |x+2| - 4$

The graph is the absolute value function that is shifted 2 units to the left and 4 units down. There are no restrictions on the domain, therefore, the domain is all real numbers.

11. $h(x) = -|x-3| + 2$

The graph is the absolute value function that is shifted 3 units to the right, reflected over the x–axis, and shifted 2 units up. There are no restrictions on the domain, therefore, the domain is all real numbers.

12. $f(x) = x^3 + 2$

The graph is the cube function that is shifted 2 units up. There are no restrictions on the domain, therefore, the domain is all real numbers.

13. $f(x) = -(x+2)^3 + 4$

The graph is the cube function that is shifted 2 units to the left, reflected over the *x*–axis, and then shifted 4 units up. There are no restrictions on the domain, therefore, the domain is all real numbers.

14. $h(x) = 3 - \sqrt[3]{x-4}$

The graph is the cube root function that is shifted 4 units to the right, reflected over the *x*–axis, and then shifted 3 units up. There are no restrictions on the domain since it has an odd root, therefore the domain is all real numbers.

In Problems 15–23 write an equation for a function that has a graph with the given characteristics.

15. The shape of $y = x^3$ shifted 6 units right.

$y = (x-6)^3$

16. The shape of $y = \sqrt{x}$ shifted 4 units down.

$y = \sqrt{x} - 4$

17. The shape of $y = |x|$ reflected over the *x*-axis and shifted 2 units up.

$y = -|x| + 2$

18. The shape of $y = x^2$ shifted 2 units right and 4 units up.

$y = (x-2)^2 + 4$

19. The shape of $y = \sqrt[3]{x}$ reflected over the *x*-axis and shifted 1 unit up.

$y = 1 - \sqrt[3]{x}$

20. The shape of $y = x^2$ reflected over the x-axis and shifted 3 units down.

$$y = -x^2 - 3$$

21. The shape of $y = \sqrt{x}$ shifted 4 units left.

$$y = \sqrt{x+4}$$

22. The shape of $y = x^3$ shifted 6 units right and 2 units down.

$$y = (x-6)^3 - 2$$

23. The shape of $y = |x|$ shifted 6 units right and 5 units up.

$$y = |x-6| + 5$$

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Section 1-3 Linear and Quadratic Functions

Goal: To describe functions that are linear and quadratic in nature

Equality Properties:

1. If $x = y$ and a is any real number, then $x \pm a = y \pm a$.

2. If $x = y$ and a is any nonzero real number, then $ax = ay$ and $\frac{x}{a} = \frac{y}{a}$.

Quadratic Functions:

Standard form of a quadratic: $f(x) = ax^2 + bx + c$ where a,b,c are real and $a \neq 0$.

Vertex form of a quadratic: $f(x) = a(x-h)^2 + k$ where $a \neq 0$ and (h,k) is the vertex.

Axis of symmetry: $x = h$

Minimum/Maximum value:

If $a > 0$, then the turning point (or vertex) is a minimum point on the graph and the minimum value would be *k*.
If $a < 0$, then the turning point (or vertex) is a maximum point on the graph and the maximum value would be *k*.

In problems 1–3, solve for the variable:

1.
$$\begin{aligned}
5x + 7 &= 9x - 13 \\
5x + 7 - 5x &= 9x - 13 - 5x \\
7 &= 4x - 13 \\
7 + 13 &= 4x - 13 + 13 \\
20 &= 4x \\
\frac{20}{4} &= \frac{4x}{4} \\
5 &= x
\end{aligned}$$

2.
$$\begin{aligned}7y+3(6y-11)&=167\\7y+18y-33&=167\\25y-33&=167\\25y-33+33&=167+33\\25y&=200\\\frac{25y}{25}&=\frac{200}{25}\\y&=8\end{aligned}$$

3.
$$\begin{aligned}\frac{m}{5}+4&=\frac{m}{2}+7\\\frac{10m}{5}+4(10)&=\frac{10m}{2}+7(10)\\2m+40&=5m+70\\40&=3m+70\\-30&=3m\\-10&=m\end{aligned}$$

For 4–11 find:
a. the domain
b. the vertex
c. the axis of symmetry
d. the x-intercept(s)
e. the y-intercept
f. the maximum or minimum value of the function

then:
g. Graph the function.
h. State the range.
i. State the interval over which the function is decreasing.
j. State the interval over which the function is increasing.

4. $f(x) = (x-1)^2 - 3$

a. The function is a quadratic, therefore the domain is all real numbers.
b. The function is in vertex form, therefore the vertex is (1, – 3).
c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = 1$.
d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = (x-1)^2 - 3$$
$$0 = x^2 - 2x + 1 - 3$$
$$0 = x^2 - 2x - 2$$

Solve using the quadratic equation

$$x = 1 \pm \sqrt{3}$$

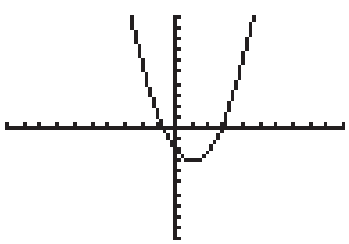
Therefore, the *x*-intercepts are $(1+\sqrt{3}, 0)$ and $(1-\sqrt{3}, 0)$.

e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = (x-1)^2 - 3$$
$$f(0) = (0-1)^2 - 3$$
$$f(0) = (-1)^2 - 3$$
$$f(0) = -2$$

Therefore, the *y*-intercept is (0, –2).

f. The graph opens upward, therefore the graph has a minimum value which is the *y*-coordinate of the vertex or –3.
g.



h. The graph has a minimum value of –3, therefore the range is $y \geq -3$.
i. Based on the graph, the function is decreasing over the interval $(-\infty, 1)$.
j. Based on the graph, the function is increasing over the interval $(1, \infty)$.

5. $f(x) = (x-2)^2 + 4$

a. The function is a quadratic, therefore the domain is all real numbers.
b. The function is in vertex form, therefore the vertex is (2, 4).
c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = 2$.
d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = (x-2)^2 + 4$$
$$0 = x^2 - 4x + 4 + 4$$
$$0 = x^2 - 4x + 8$$

Solving the above equation by the quadratic equation will result in complex roots, therefore no *x*-intercepts are present.

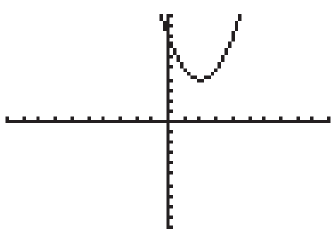
e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = (x-2)^2 + 4$$
$$f(0) = (0-2)^2 + 4$$
$$f(0) = (-2)^2 + 4$$
$$f(0) = 8$$

Therefore, the *y*-intercept is (0, 8).

f. The graph opens upward, therefore the graph has a minimum value which is the *y*-coordinate of the vertex or 4.

g.



h. The graph has a minimum value of 4, therefore the range is $y \geq 4$.
i. Based on the graph, the function is decreasing over the interval $(-\infty, 2)$.
j. Based on the graph, the function is increasing over the interval $(2, \infty)$.

6. $f(x) = -x^2 + 7$

a. The function is a quadratic, therefore the domain is all real numbers.

b. The function in vertex form is $f(x) = -(x-0)^2 + 7$, therefore the vertex is (0, 7).

c. The axis of symmetry is the x–value of the vertex, therefore the axis of symmetry is $x = 0$.

d. The x-intercepts are found by setting $f(x) = 0$.

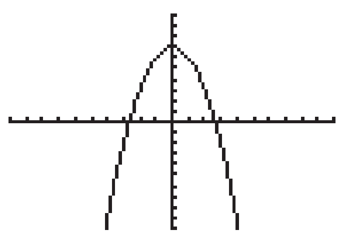
$$f(x) = -x^2 + 7$$
$$0 = -x^2 + 7$$
Solve using the quadratic equation
$$x = \pm\sqrt{7}$$
Therefore, the x-intercepts are $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$.

e. The y-intercepts are found by setting $x = 0$.

$$f(x) = -x^2 + 7$$
$$f(0) = -0^2 + 7$$
$$f(0) = 0 + 7$$
$$f(0) = 7$$
Therefore, the y-intercept is (0, 7).

f. The graph opens downward, therefore the graph has a maximum value which is the y-coordinate of the vertex or 7.

g.



h. The graph has a maximum value of 2, therefore the range is $y \leq 0$.

i. Based on the graph, the function is decreasing over the interval $(0, \infty)$.

j. Based on the graph, the function is increasing over the interval $(-\infty, 0)$.

7. $f(x) = -(x-1)^2 - 1$

a. The function is a quadratic, therefore the domain is all real numbers.
b. The function is in vertex form, therefore the vertex is (1, –1).
c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = 1$.
d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = -(x-1)^2 - 1$$
$$0 = -x^2 + 2x - 1 - 1$$
$$0 = -x^2 + 2x - 2$$

Solving the above equation by the quadratic equation will result in complex roots, therefore no *x*-intercepts are present.

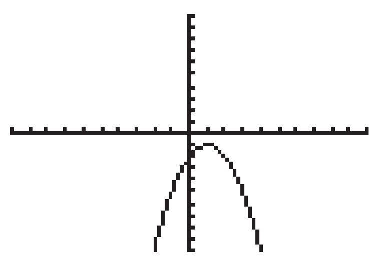
e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = -(x-1)^2 - 1$$
$$f(0) = -(0-1)^2 - 1$$
$$f(0) = -(-1)^2 - 1$$
$$f(0) = -2$$

Therefore, the *y*-intercept is (0, –2).

f. The graph opens downward, therefore the graph has a maximum value which is the *y*-coordinate of the vertex or –1.

g.



h. The graph has a maximum value of –1, therefore the range is $y \le -1$.
i. Based on the graph, the function is decreasing over the interval $(1, \infty)$.
j. Based on the graph, the function is increasing over the interval $(-\infty, 1)$.

8. $f(x) = x^2 - 4x$

a. The function is a quadratic, therefore the domain is all real numbers.
b. The function in vertex form is $f(x) = (x-2)^2 - 4$, therefore the vertex is (2, – 4).
c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = 2$.
d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = x^2 - 4x$$
$$0 = x^2 - 4x$$
$$0 = x(x-4)$$
$$x = 0, 4$$

Therefore, the *x*-intercepts are $(0,0)$ and $(4,0)$.

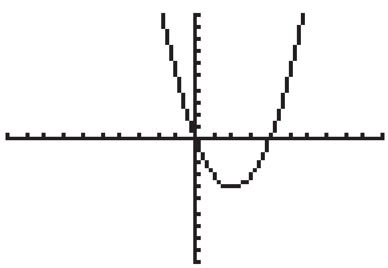
e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = x^2 - 4x$$
$$f(0) = 0^2 - 4(0)$$
$$f(0) = 0 + 0$$
$$f(0) = 0$$

Therefore, the *y*-intercept is (0, 0).

f. The graph opens upward, therefore the graph has a minimum value which is the *y*-coordinate of the vertex or – 4.

g.



h. The graph has a minimum value of – 4, therefore the range is $y \geq -4$.
i. Based on the graph, the function is decreasing over the interval $(-\infty, 2)$.
j. Based on the graph, the function is increasing over the interval $(2, \infty)$.

9. $f(x) = x^2 + 2x - 4$

a. The function is a quadratic, therefore the domain is all real numbers.

b. The function in vertex form is $f(x) = (x+1)^2 - 5$, therefore the vertex is (–1, –5).

c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = -1$.

d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = (x+1)^2 - 5$$
$$0 = x^2 + 2x + 1 - 5$$
$$0 = x^2 + 2x - 4$$

Solve using the quadratic equation

$$x = -1 \pm \sqrt{5}$$

Therefore, the *x*-intercepts are $(-1+\sqrt{5}, 0)$ and $(-1-\sqrt{5}, 0)$.

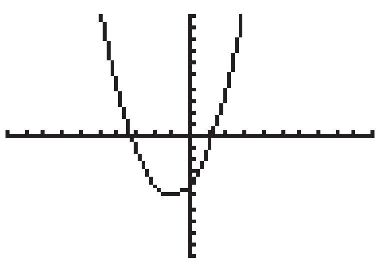
e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = (x+1)^2 - 5$$
$$f(0) = (0+1)^2 - 5$$
$$f(0) = (1)^2 - 5$$
$$f(0) = -4$$

Therefore, the *y*-intercept is (0, –4).

f. The graph opens upward, therefore the graph has a minimum value which is the *y*-coordinate of the vertex or –5.

g.



h. The graph has a minimum value of –5, therefore the range is $y \geq -5$.

i. Based on the graph, the function is decreasing over the interval $(-\infty, -1)$.

j. Based on the graph, the function is increasing over the interval $(-1, \infty)$.

10. $f(x) = x^2 + 2x + 1$

a. The function is a quadratic, therefore the domain is all real numbers.

b. The function in vertex form is $f(x) = (x+1)^2 + 0,$ therefore the vertex is (–1, 0).

c. The axis of symmetry is the *x*–value of the vertex, therefore the axis of symmetry is $x = -1$.

d. The *x*-intercepts are found by setting $f(x) = 0$.

$$f(x) = x^2 + 2x + 1$$
$$0 = x^2 + 2x + 1$$
$$0 = (x+1)(x+1)$$
$$x = -1$$

Therefore, there is only one *x*-intercept which is $(-1, 0)$.

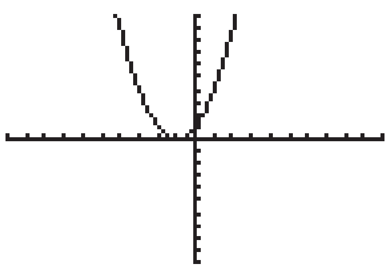
e. The *y*-intercepts are found by setting $x = 0$.

$$f(x) = x^2 + 2x + 1$$
$$f(0) = 0^2 + 2(0) + 1$$
$$f(0) = 0 + 0 + 1$$
$$f(0) = 1$$

Therefore, the *y*-intercept is (0, 1).

f. The graph opens upward, therefore the graph has a minimum value which is the *y*-coordinate of the vertex or 0.

g.



h. The graph has a minimum value of 0, therefore the range is $y \geq 0$.

i. Based on the graph, the function is decreasing over the interval $(-\infty, -1)$.

j. Based on the graph, the function is increasing over the interval $(-1, \infty)$.

11. $f(x) = -x^2 + 10x - 19$

a. The function is a quadratic, therefore the domain is all real numbers.

b. The function in vertex form is $f(x) = -(x-5)^2 + 6$, therefore the vertex is (5, 6).

c. The axis of symmetry is the x–value of the vertex, therefore the axis of symmetry is $x = 5$.

d. The x-intercepts are found by setting $f(x) = 0$.

$$f(x) = -x^2 + 10x - 19$$
$$0 = -x^2 + 10x - 19$$
$$0 = x^2 - 10x + 19$$

Solve using the quadratic equation

$$x = 5 \pm \sqrt{6}$$

Therefore, the x-intercepts are $(5+\sqrt{6}, 0)$ and $(5-\sqrt{6}, 0)$.

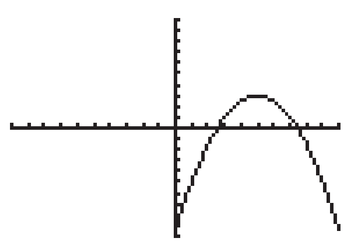
e. The y-intercepts are found by setting $x = 0$.

$$f(x) = -x^2 + 10x - 19$$
$$f(0) = -0^2 + 10(0) - 19$$
$$f(0) = 0 + 0 - 19$$
$$f(0) = -19$$

Therefore, the y-intercept is (0, –19).

f. The graph opens downward, therefore the graph has a maximum value which is the y-coordinate of the vertex or 6.

g.



h. The graph has a maximum value of 4, therefore the range is $y \le 6$.

i. Based on the graph, the function is decreasing over the interval $(5, \infty)$.

j. Based on the graph, the function is increasing over the interval $(-\infty, 5)$.

12. The revenue and cost functions for a company that manufactures components for washing machines were determined to be:

$$R(x) = x(200 - 4x) \qquad \text{and} \qquad C(x) = 160 + 20x$$

where x is the number of components in millions and $R(x)$ and $C(x)$ are in millions of dollars.

a) How many components must be sold in order for the company to break even? (Break-even points are when $R(x) = C(x)$.) (Round answers to nearest million.)

$$R(x) = C(x)$$
$$x(200 - 4x) = 160 + 20x$$
$$200x - 4x^2 = 160 + 20x$$
$$0 = 4x^2 - 180x + 160$$

Solve the equation by the quadratic equation

$$x \approx 0.9, 44.09$$

The company would need to sell approximately 1 million or 44 million to break even.

b) Find the profit equation. ($P(x) = R(x) - C(x)$)

$$P(x) = R(x) - C(x)$$
$$P(x) = x(200 - 4x) - (160 + 20x)$$
$$P(x) = 200x - 4x^2 - 160 - 20x$$
$$P(x) = -4x^2 + 180x - 160$$

c) Determine the maximum profit. How many components must be sold in order to achieve that maximum profit?

The maximum profit occurs at the vertex of the profit function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{180}{2(-4)} = \frac{-180}{-8} = 22.5.$ To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$P(x) = -4x^2 + 180x - 160$$
$$P(22.5) = -4(22.5)^2 + 180(22.5) - 160$$
$$P(22.5) = -2025 + 4050 - 160$$
$$P(22.5) = 1865$$

The maximum profit of \$1,865 million is achieved when 22.5 million components are sold.

13. A company keeps records of the total revenue (money taken in) in thousands of dollars from the sale of x units (in thousands) of a product. It determines that total revenue is a function *R(x)* given by

$$R(x) = 300x - x^2$$

It also keeps records of the total cost of producing x units of the same product. It determines that the total cost is a function $C(x)$ given by

$$C(x) = 40x + 1600$$

a) Find the break-even points for this company. (Round answer to nearest 1000.)

$$R(x) = C(x)$$
$$300x - x^2 = 40x + 1600$$
$$0 = x^2 - 260x + 1600$$

Solve the equation by the quadratic equation

$$x \approx 6.307, 253.693$$

The company would need to sell approximately 6,000 or 254,000 to break even.

b) Determine at what point profit is at a maximum. What is the maximum profit? How many units must be sold in order to achieve maximum profit?

The profit equation is:

$$P(x) = R(x) - C(x)$$
$$P(x) = 300x - x^2 - (40x + 1600)$$
$$P(x) = -x^2 + 260x - 1600$$

The maximum profit occurs at the vertex of the profit function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{260}{2(-1)} = \frac{-260}{-2} = 130$. To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$P(x) = -x^2 + 260x - 1600$$
$$P(130) = -(130)^2 + 260(130) - 1600$$
$$P(130) = -16,900 + 33,800 - 1600$$
$$P(130) = 15,300$$

The maximum profit of \$15,300 thousands or \$15,300,000 is achieved when 130,000 units are sold.

14. The cost, *C(x)*, of building a house is a function of the number of square feet, *x*, in the house. If the cost function can be approximated by

$$C(x) = 0.01x^2 - 20x + 25,000 \qquad \text{where } 1000 \le x \le 3500$$

a) What would be the cost of building a 1500 square foot house?

Substitute the value of 1500 into the cost function:

$$C(x) = 0.01x^2 - 20x + 25,000$$
$$C(1500) = 0.01(1500)^2 - 20(1500) + 25,000$$
$$C(1500) = 0.01(2,250,000) - 30,000 + 25,000$$
$$C(1500) = 17,500$$

It will cost $17,500 to build a 1500 square foot house.

b) Find the minimum cost to build a house. How many square feet would that house have?

The minimum cost occurs at the vertex of the cost function. The *x*-coordinate is $x = -\frac{b}{2a} = -\frac{-20}{2(0.01)} = \frac{20}{0.02} = 1000$. To find the *y*-coordinate of the vertex, substitute the value into the function as follows:

$$C(x) = 0.01x^2 - 20x + 25000$$
$$C(1000) = 0.01(1000)^2 - 20(1000) + 25,000$$
$$C(1000) = 10,000 - 20,000 + 25,000$$
$$C(1000) = 15,000$$

The minimum cost of $15,000 is achieved when a 1000 square foot home is built.

15. The cost of producing computer software is a function of the number of hours worked by the employees. If the cost function can be approximated by

$$C(x) = 0.04x^2 - 20x + 6000 \qquad \text{where } 200 \le x \le 1000$$

a) What would be the cost if the employees worked 800 hours?

Substitute the value of 800 into the cost function:

$$C(x) = 0.04x^2 - 20x + 6000$$
$$C(800) = 0.04(800)^2 - 20(800) + 6000$$
$$C(800) = 0.04(640,000) - 16,000 + 6000$$
$$C(800) = 15,600$$

If the employees work 800 hours, it will cost $15,600 to produce the software.

b) Find the number of hours the employees should work in order to minimize the cost. What would the minimum cost be?

The minimum cost occurs at the vertex of the cost function. The x-coordinate is $x = -\frac{b}{2a} = -\frac{-20}{2(0.04)} = \frac{20}{0.08} = 250.$ To find the y-coordinate of the vertex, substitute the value into the function as follows:

$$C(x) = 0.04x^2 - 20x + 6000$$
$$C(250) = 0.04(250)^2 - 20(250) + 6000$$
$$C(250) = 2500 - 5000 + 6000$$
$$C(250) = 3500$$

The minimum cost of $3500 is achieved when the employees work 250 hours.

16. *Break-even Analysis*. A publisher for a promising new novel figures fixed costs (overhead, advances, promotion, copy, editing, typesetting, and so on) at $67,000 and variable costs (printing, paper, binding, shipping) at $3.50 for each book produced. If the book is sold to distributors for $21 each, how many must be produced and sold for the publisher to break even?

Let x = the number of books produced. Since the break even point is the point when cost is the same as the revenue:

$$21x = 67,000 + 3.50x$$
$$17.50x = 67,000$$
$$x = 3828.571429$$

Therefore, the publisher must produce 3829 books to break even.

Name ________________________________ **Date** ______________ **Class** ____________

Section 1-4 Polynomial and Rational Functions

Goal: To describe and identify functions that are polynomial and rational in nature

Definition: Polynomial function

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ for n a nonnegative integer, called the degree of the polynomial. The coefficients $a_0, a_1, \ldots, a_n$ are real numbers with $a_n \neq 0$. The domain of a polynomial function is the set of all real numbers.

Definition: Rational function

$f(x) = \dfrac{n(x)}{d(x)}$ $d(x) \neq 0$ where *n(x)* and *d(x)* are polynomials. The domain is the set of all real numbers such that $d(x) \neq 0$.

<u>Vertical Asymptotes:</u>

<u>Case 1</u>: Suppose *n(x)* and *d(x)* have no real zero in common. If *c* is a real number such that $d(x) = 0,$ then the line $x = c$ is a vertical asymptote of the graph.
<u>Case 2</u>: If *n(x)* and *d(x)* have one or more real zeros in common, cancel common linear factors and apply Case 1 to the reduced fraction.

<u>Horizontal Asymptotes</u>:

<u>Case 1</u>: If degree *n(x)* < degree *d(x)*, then $y = 0$ is the horizontal asymptote.
<u>Case 2</u>: If degree *n(x)* = degree *d(x)*, then $y = a/b$ is the horizontal asymptote, where *a* is the leading coefficient of *n(x)* and *b* is the leading coefficient of *d(x)*.
<u>Case 3</u>: If degree *n(x)* > degree *d(x)*, there is no horizontal asymptote.

For 1–6 determine each of the following for the polynomial functions:
a. the degree of the polynomial
b. the *x*–intercept(s) of the graph of the polynomial
c. the *y*–intercept of the graph of the polynomial

1. $f(x) = x^3 - 7x - 6 = (x+2)(x-3)(x+1)$

   a. The degree of the polynomial is the highest exponent, which is 3.
   b. The x–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore if we set the factors equal to zero, the zeros of the polynomial occur at –1, –2, and 3.
   c. The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore the y-intercept is (0, –6).

2. $f(x) = x^3 + 4x^2 - 4x - 16 = (x-2)(x+2)(x+4)$

   a. The degree of the polynomial is the highest exponent, which is 3.
   b. The x–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore if we set the factors equal to zero, the zeros of the polynomial occur at 2, –2, and –4.
   c. The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore the y-intercept is (0, –16).

3. $f(x) = x^3 - 3x^2 - 10x + 24 = (x+3)(x-2)(x-4)$

   a. The degree of the polynomial is the highest exponent, which is 3.
   b. The x–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore if we set the factors equal to zero, the zeros of the polynomial occur at –3, 2, and 4.
   c. The y-intercept occurs when the x value is zero. If the x value is zero, the only term that will not be zero is the constant term, therefore the y-intercept is (0, 24).

4. $f(x) = x^3 + 4x^2 - x - 4 = (x+4)(x+1)(x-1)$

a. The degree of the polynomial is the highest exponent, which is 3.
b. The *x*–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form, therefore if we set the factors equal to zero, the zeros of the polynomial occur at –4, –1, and 1.
c. The *y*-intercept occurs when the *x* value is zero. If the *x* value is zero, the only term that will not be zero is the constant term, therefore the *y*-intercept is (0, –4).

5. $f(x) = x^4 - 2x^3 + x^2 + 2x - 2 = (x-1)(x+1)(x^2 - 2x + 2)$

a. The degree of the polynomial is the highest exponent, which is 4.
b. The *x*–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form. The third factor must be solved using the quadratic equation, therefore, the zeros of the polynomial occur at 1, –1, and $1 \pm \sqrt{3}$.
c. The *y*-intercept occurs when the *x* value is zero. If the *x* value is zero, the only term that will not be zero is the constant term, therefore the *y*-intercept is (0, –2).

6. $f(x) = x^5 + 5x^4 - 20x^2 - x + 15 = (x+3)(x-1)(x+1)(x^2 + 2x - 5)$

a. The degree of the polynomial is the highest exponent, which is 5.
b. The *x*–intercept(s) are found by setting the polynomial equal to zero. The polynomial is already in factored form. The fourth factor must be solved using the quadratic equation, therefore, the zeros of the polynomial occur at –3, –1, 1, and $-1 \pm \sqrt{6}$.
c. The *y*-intercept occurs when the *x* value is zero. If the *x* value is zero, the only term that will not be zero is the constant term, therefore the *y*-intercept is (0, –15).

For the given rational functions in 7–12:
a. Find the domain.
b. Find any x-intercept(s).
c. Find any y-intercept.
d. Find any vertical asymptote.
e. Find any horizontal asymptote.
f. Sketch a graph of $y = f(x)$ for $-10 \le x \le 10$.

7. $f(x) = \dfrac{1}{2x}$

a. The function is defined everywhere except when the denominator is zero. The domain is therefore all real numbers except 0.

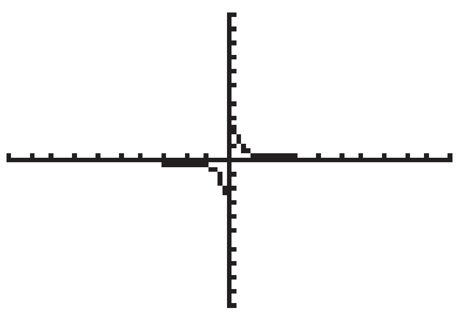
b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression and the numerator cannot be zero, the function value cannot be zero. Therefore, there is no x–intercept.

c. The y–intercept is found when the value of x is zero. Since $x = 0$ is not in the domain, there is no y–intercept.

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 0$, the vertical asymptote is the line $x = 0$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{1}{2x} = \dfrac{\frac{1}{2x}}{\frac{2x}{2x}} = \dfrac{\frac{1}{2x}}{1}$, as x increases or decreases without bound, the denominator is always 1 and the numerator tends to 0; so $f(x)$ tends to 0. The horizontal asymptote is the line $y = 0$.

f.



8. $f(x) = \dfrac{3x}{x-5}$

a. The function is defined everywhere except when the denominator is zero.

$$x - 5 = 0$$
$$x = 5$$

The domain is therefore all real numbers except 5.

b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$3x = 0$$
$$x = 0$$

Therefore, the x–intercept is (0, 0).

c. The y–intercept is found when the value of x is zero.

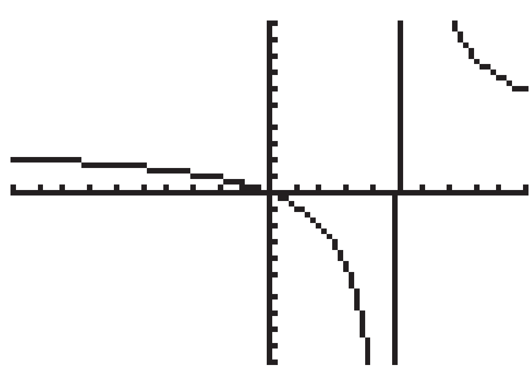
$$f(x) = \frac{3x}{x-5}$$
$$f(0) = \frac{3(0)}{0-5}$$
$$f(0) = \frac{0}{-5} = 0$$

Therefore, the y–intercept is (0, 0).

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 5$, the vertical asymptote is the line $x = 5$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{3x}{x-5} = \dfrac{\frac{3x}{x}}{\frac{x}{x} - \frac{5}{x}} = \dfrac{3}{1 - \frac{5}{x}}$, as x increases or decreases without bound, the numerator is always 3 and the denominator tends to 1 – 0, or 1; so $f(x)$ tends to 3. The horizontal asymptote is the line $y = 3$.

f.



9. $f(x) = \dfrac{3x}{x-3}$

a. The function is defined everywhere except when the denominator is zero.

$$x - 3 = 0$$
$$x = 3$$

The domain is therefore all real numbers except 3.

b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$3x = 0$$
$$x = 0$$

Therefore, the x–intercept is (0, 0).

c. The y–intercept is found when the value of x is zero.

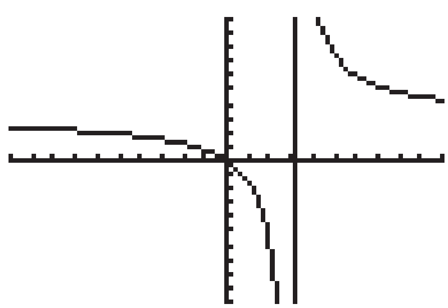
$$f(x) = \frac{3x}{x-3}$$
$$f(0) = \frac{3(0)}{0-3}$$
$$f(0) = \frac{0}{-3} = 0$$

Therefore, the y–intercept is (0, 0).

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 3$, the vertical asymptote is the line $x = 3$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x) = \dfrac{3x}{x-3} = \dfrac{\frac{3x}{x}}{\frac{x}{x} - \frac{3}{x}} = \dfrac{3}{1 - \frac{3}{x}}$, as x increases or decreases without bound, the numerator is always 3 and the denominator is tends to 1 – 0, or 1; so $f(x)$ tends to 3. The horizontal asymptote is the line $y = 3$.

f.



10. $f(x)=\dfrac{2x-4}{x+3}$

a. The function is defined everywhere except when the denominator is zero.

$$x+3=0$$
$$x=-3$$

The domain is therefore all real numbers except –3.

b. The *x*–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$2x-4=0$$
$$2x=4$$
$$x=2$$

Therefore, the *x*–intercept is (2, 0).

c. The *y*–intercept is found when the value of *x* is zero.

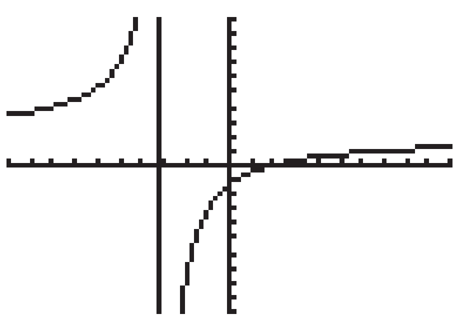
$$f(x)=\frac{2x-4}{x+3}$$
$$f(0)=\frac{2(0)-4}{0+3}$$
$$f(0)=\frac{-4}{3}$$

Therefore, the *y*–intercept is $(0,-\frac{4}{3})$.

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x=-3$, the vertical asymptote is the line $x=-3$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of *x*. Therefore, $f(x)=\dfrac{2x-4}{x+3}=\dfrac{\frac{2x}{x}-\frac{4}{x}}{\frac{x}{x}+\frac{3}{x}}=\dfrac{2-\frac{4}{x}}{1+\frac{3}{x}}$, as *x* increases or decreases without bound, the numerator tends to 2 – 0, or 2 and the denominator tends to 1 – 0, or 1; so *f*(*x*) tends to 2. The horizontal asymptote is the line $y=2$.

f.



11. $f(x)=\dfrac{4+x}{4-x}$

a. The function is defined everywhere except when the denominator is zero.

$$\begin{aligned}4-x&=0\\4&=x\end{aligned}$$

The domain is therefore all real numbers except 4.

b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, the function value is zero when the numerator is zero.

$$\begin{aligned}4+x&=0\\x&=-4\end{aligned}$$

Therefore, the x–intercept is (–4, 0).

c. The y–intercept is found when the value of x is zero.

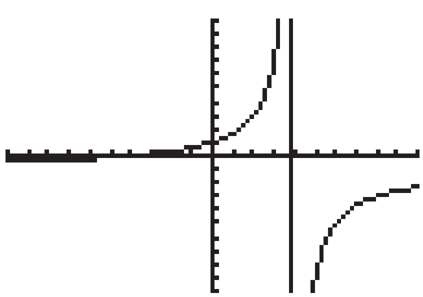
$$\begin{aligned}f(x)&=\frac{4+x}{4-x}\\f(0)&=\frac{4+0}{4-0}\\f(0)&=\frac{4}{4}=1\end{aligned}$$

Therefore, the y–intercept is $(0,1)$.

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x = 4$, the vertical asymptote is the line $x = 4$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x)=\dfrac{4+x}{4-x}=\dfrac{\frac{4}{x}+\frac{x}{x}}{\frac{4}{x}-\frac{x}{x}}=\dfrac{\frac{4}{x}+1}{\frac{4}{x}-1}$, as x increases or decreases without bound, the numerator tends to 0 + 1, or 1 and the denominator tends to 0 – 1, or –1; so $f(x)$ tends to –1. The horizontal asymptote is the line $y=-1$.

f.



12. $f(x)=\frac{1-5x}{1+2x}$

a. The function is defined everywhere except when the denominator is zero.

$$\begin{aligned}1+2x&=0\\2x&=-1\\x&=-\frac{1}{2}\end{aligned}$$

The domain is therefore all real numbers except $-\frac{1}{2}$.

b. The x–intercept(s) are found by setting the function equal to 0. Since the function is a rational expression, when the numerator is zero, the function value is zero.

$$\begin{aligned}1-5x&=0\\-5x&=-1\\x&=\frac{1}{5}\end{aligned}$$

Therefore, the x–intercept is $(\frac{1}{5},0)$.

c. The y–intercept is found when the value of x is zero.

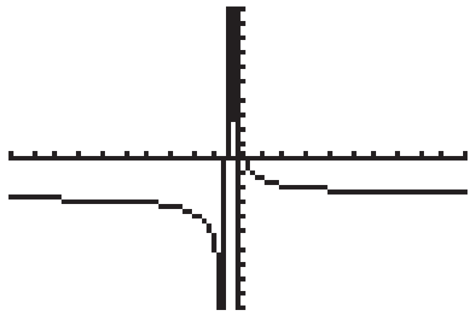
$$\begin{aligned}f(x)&=\frac{1-5x}{1+2x}\\f(0)&=\frac{1-5(0)}{1+2(0)}\\f(0)&=\frac{1}{1}=1\end{aligned}$$

Therefore, the y–intercept is $(0,1)$.

d. Vertical asymptotes occur at values where the function is not defined. Since the function is not defined for $x=-\frac{1}{2}$, the vertical asymptote is the line $x=-\frac{1}{2}$.

e. Horizontal asymptotes are found by dividing all terms by the highest power of x. Therefore, $f(x)=\frac{1-5x}{1+2x}=\frac{\frac{1}{x}-\frac{5x}{x}}{\frac{1}{x}+\frac{2x}{x}}=\frac{\frac{1}{x}-5}{\frac{1}{x}+2}$, as x increases or decreases without bound, the numerator tends to 0 – 5, or –5 and the denominator tends to 0 + 2, or 2; so $f(x)$ tends to $-\frac{5}{2}$. The horizontal asymptote is the line $y=-\frac{5}{2}$.

f.



13. A video production company is planning to produce a documentary. The producer estimates that it will cost \$52,000 to produce the video and \$20 per video to copy and distribute the tape.

a) Assuming that the total cost to market the video, *C(n),* is linearly related to the total number, *n*, of videos produced, write an equation for the cost function.

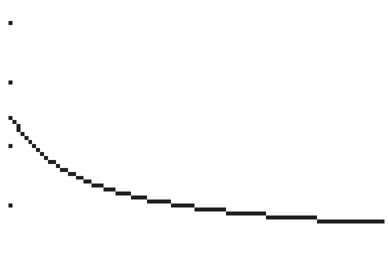
$$C(n) = 20n + 52{,}000$$

b) The average cost per video for an output of *n* videos is given by $\overline{C}(n) = \dfrac{C(n)}{n}$.

Find the average cost function. $\overline{C}(n) = \dfrac{C(n)}{n} = \dfrac{20n + 52{,}000}{n}$

c) Sketch a graph of the average cost function for $500 \le n \le 3000$.

The *x*–axis scale shown is from 500 to 3000. Each tick mark is 500 units. The *y*–axis scale shown is from 0 to 200. Each tick mark is 50 units.



d) What does the average cost per video tend to as production increases?

To find the value that the function tends to go towards, you will find the horizontal asymptote of the function. Horizontal asymptotes are found by dividing all terms by the highest power of *n*. Therefore, $\overline{C}(n) = \dfrac{\frac{20n}{n} + \frac{52{,}000}{n}}{\frac{n}{n}} = \dfrac{20 + \frac{52{,}000}{n}}{1}$, as *n* increases without bound, the numerator tends to 20 + 0, or 20 and the denominator is always 1; so $\overline{C}(n)$ tends to 20. This means that the average cost per video tends towards \$20 each.

14. A contractor purchases a piece of equipment for \$36,000. The equipment requires an average expenditure of \$8.25 per hour for fuel and maintenance, and the operator is paid \$13.50 per hour to operate the machinery.

a) Assuming that the total cost per day, *C(h)*, is linearly related to the number of hours, *h*, that the machine is operated, write an equation for the cost function.

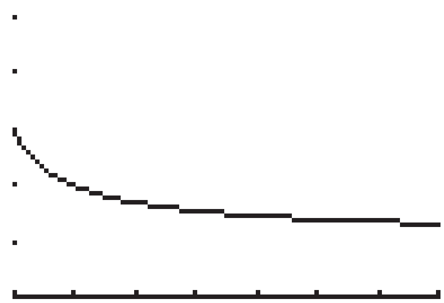
$$C(h) = 21.75h + 36,000$$

b) The average cost per hour of operating the machine is given by $\overline{C}(h) = \frac{C(h)}{h}$.

Find the average cost function. $\overline{C}(h) = \frac{C(h)}{h} = \frac{21.75h + 36,000}{h}$

c) Sketch a graph of the average cost function for $1000 \le h \le 8000$.

The *x*–axis scale shown is from 1000 to 8000. Each tick mark is 1000 units. The *y*–axis scale shown is from 0 to 100. Each tick mark is 20 units.



d) What cost per hour does the average cost per hour tend to as the number of hours of use increases?

To find the value that the function tends to go towards, you will find the horizontal asymptote of the function. Horizontal asymptotes are found by dividing all terms by the highest power of *h*. Therefore, $\overline{C}(h) = \frac{\frac{21.75n}{h} + \frac{36,000}{h}}{\frac{h}{h}} = \frac{21.75 + \frac{36,000}{h}}{1}$, as *h* increases without bound, the numerator tends to 21.75 + 0, or 21.75 and the denominator is always 1; so $\overline{C}(h)$ tends to 21.75. This means that the average cost per hour tends towards \$21.75 as the number of hours of use increases.

15. The daily cost function for producing x printers for home computers was determined to be:

$$C(x) = x^2 + 8x + 6000$$

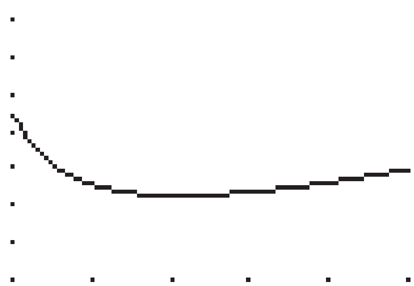
The average cost per printer at a production level of x printers per day is $\overline{C}(x) = \frac{C(x)}{x}$.

a) Find the average cost function.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{x^2 + 8x + 6000}{x}$$

b) Sketch a graph of the average cost function for $25 \le x \le 150$.

The x–axis scale shown is from 25 to 150. Each tick mark is 25 units. The y–axis scale shown is from 50 to 400. Each tick mark is 50 units.



c) At what production level is the daily average cost at a minimum? What is that minimum value?

Based on the graph above, the minimum value occurs around the third tick mark, which has a value of 75. By substituting values into the average value equation, we would have the following:

$$\overline{C}(75) = 163$$
$$\overline{C}(76) = 162.947$$
$$\overline{C}(77) = 162.922$$
$$\overline{C}(78) = 162.923$$
$$\overline{C}(79) = 162.949$$

Therefore, the minimum average cost of 162.92 occurs when 77 printers are produced.

16. The monthly cost function for producing x brake assemblies for a certain type of car is given by:

$$C(x) = 3x^2 + 36x + 9000$$

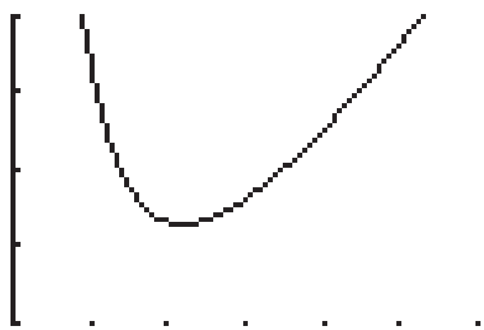
The average cost per brake assembly at a production level of x assemblies per month is $\overline{C}(x) = \dfrac{C(x)}{x}$.

a) Find the average cost function.

$$\overline{C}(x) = \frac{C(x)}{x} = \frac{3x^2 + 36x + 9000}{x}$$

b) Sketch a graph of the average cost function for $0 \le x \le 150$.

The x–axis scale shown is from 0 to 150. Each tick mark is 25 units. The y–axis scale shown is from 300 to 500. Each tick mark is 50 units.



c) At what production level is the daily average cost at a minimum? What is that minimum value?

Based on the graph above, the minimum value occurs just beyond the third tick mark, which has a value of 50. By substituting values into the average value equation, we would have the following:

$$\overline{C}(53) = 364.811$$
$$\overline{C}(54) = 364.667$$
$$\overline{C}(55) = 364.636$$
$$\overline{C}(56) = 364.714$$

Therefore, the minimum average cost of 364.63 occurs when 55 brake assemblies are produced.

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Section 1-5 Exponential Functions

Goal: To describe and solve functions that are exponential in nature

Rules for Exponents:

$a^m \cdot a^n = a^{m+n}$ Product Rule $a^0 = 1,\ \ a \neq 0$ Zero Exponent Rule

$\dfrac{a^m}{a^n} = a^{m-n}$ Quotient Rule $\left(a^m\right)^n = a^{mn}$ Power Rule

In problems 1–8, describe in words the transformations that can be used to obtain the graph of $g(x)$ from the graph of $f(x)$.

1. $g(x) = 4^{x+3} - 4;\ \ f(x)=4^x$

   The function f is shifted 3 units to the left and 4 units down.

2. $g(x) = -3^x - 5;\ \ f(x) = 3^x$

   The function f is reflected over the x–axis and shifted 5 units down.

3. $g(x) = 2^{x-4} - 6;\ \ f(x) = 2^x$

   The function f is shifted 4 units to the right and 6 units down.

4. $g(x) = -5^{x-3} + 2;\ \ f(x) = 5^x$

   The function f is shifted 3 units to the right, reflected over the x–axis, and shifted 2 units up.

5. $g(x) = 10^{x-2} - 5;\quad f(x) = 10^x$

The function *f* is shifted 2 units to the right and 5 units down.

6. $g(x) = -10^x - 3;\quad f(x) = 10^x$

The function *f* is reflected over the *x*–axis and shifted 3 units down.

7. $g(x) = e^{x+1} + 2;\quad f(x) = e^x$

The function *f* is shifted 1 unit to the left and 2 units up.

8. $g(x) = -e^x + 5;\quad f(x) = e^x$

The function *f* is reflected over the *x*–axis and shifted 5 units up.

In Problems 9–20, solve each equation for *x*.

9. $10^{2x-3} = 10^{5x+4}$

$$\begin{aligned} 2x-3 &= 5x+4 \\ -7 &= 3x \\ -\frac{7}{3} &= x \end{aligned}$$

10. $10^{x^2} = 10^{2x+8}$

$$\begin{aligned} x^2 &= 2x+8 \\ x^2-2x-8 &= 0 \\ (x-4)(x+2) &= 0 \\ x &= -2, 4 \end{aligned}$$

11. $6^{5x-4} = 6^{x^2}$

$$\begin{aligned} 5x-4 &= x^2 \\ x^2-5x+4 &= 0 \\ (x-4)(x-1) &= 0 \\ x &= 1, 4 \end{aligned}$$

12. $8^{x^2} = 8^{8x}$

$$\begin{aligned} x^2 &= 8x \\ x^2-8x &= 0 \\ x(x-8) &= 0 \\ x &= 0, 8 \end{aligned}$$

13. $(x+6)^3 = (3x-8)^3$

$$\begin{aligned} x+6 &= 3x-8 \\ 14 &= 2x \\ 7 &= x \end{aligned}$$

14. $(2x-7)^5 = (x+1)^5$

$$\begin{aligned} 2x-7 &= x+1 \\ x &= 8 \end{aligned}$$

15. $(e^3)^4 = e^x$

$$3(4) = x$$
$$12 = x$$

16. $(e^x)^4 = e^{x^2}$

$$4x = x^2$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0, 4$$

17. $(e^{2x})^x = e^{15+x}$

$$2x^2 = 15 + x$$
$$2x^2 - x - 15 = 0$$
$$(2x+5)(x-3) = 0$$
$$x = -\tfrac{5}{2}, 3$$

18. $3^x \cdot 3^4 = 3^{3x^2}$

$$x + 4 = 3x^2$$
$$3x^2 - x - 4 = 0$$
$$(3x-4)(x+1) = 0$$
$$x = \tfrac{4}{3}, -1$$

19. $2^{x^2} = 2^{12x} \cdot 2^{-32}$

$$x^2 = 12x - 32$$
$$x^2 - 12x + 32 = 0$$
$$(x-4)(x-8) = 0$$
$$x = 4, 8$$

20. $9^x \cdot 9 = 9^{2x^2}$

$$x + 1 = 2x^2$$
$$2x^2 - x - 1 = 0$$
$$(2x+1)(x-1) = 0$$
$$x = -\tfrac{1}{2}, 1$$

INTEREST FORMULAS

Simple Interest:	$A = P(1 + rt)$
Compound Interest:	$A = P\left(1 + \frac{r}{m}\right)^{mt}$
Continuous Compound Interest:	$A = Pe^{rt}$

where P is the amount invested (principal), r (expressed as a decimal) is the annual interest rate, t is time invested (in years), m is the number of times a year the interest is compounded, and A is the amount of money in the account after t years (future value).

(Round answers for 21 –28 to the nearest dollar)

21. Fred inherited $35,000 from his uncle. He decides to invest his money for 5 years in order to have the greatest down-payment when he buys a house. He can choose from 3 different banks.

Bank A offers 1% compounded monthly.
Bank B offers .5% compounded continuously.
Bank C offers .75% compounded daily.

Which bank offers the best plan so Fred can earn the most money from his investment?

Bank A:

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 35,000\left(1+\frac{0.01}{12}\right)^{12(5)}$$
$$A = 35000(1.000833)^{60}$$
$$A \approx \$36,794$$

Bank B:

$$A = Pe^{rt}$$
$$A = 35,000e^{(0.005)(5)}$$
$$A = 35,000e^{0.025}$$
$$A \approx \$35,886$$

Bank C:

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 35,000\left(1+\frac{0.0075}{365}\right)^{365(5)}$$
$$A = 35,000(1.000021)^{1825}$$
$$A \approx \$36,337$$

Therefore, Bank A is the best option.

22. The day your first child is born you invest $10,000 in an account that pays 1.2% interest compounded quarterly. How much will be in the account when the child is 18 years old and ready to start to college?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 10,000\left(1+\frac{0.012}{4}\right)^{4(18)}$$
$$A = 10,000(1.003)^{72}$$
$$A \approx \$12,407$$

23. When your second child is born, you are able to invest only \$5000 but the account pays 1% interest compounded daily. How much will be in the account when this child is 18 years old and ready to start to college?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 5000\left(1+\frac{0.01}{365}\right)^{365(18)}$$
$$A = 5000(1.000027)^{6570}$$
$$A \approx \$5986$$

24. When your third child comes along, money is even tighter and you are able to invest only \$1000, but you are able to find a bank that will let you invest the money at 1.75% compounded continuously. How much will be in the account when this third child is 18 years old and ready to start to college?

$$A = Pe^{rt}$$
$$A = 1000e^{(0.0175)(18)}$$
$$A = 1000e^{0.315}$$
$$A \approx \$1370$$

25. Joe Vader plans to start his own business in ten years. How much money would he need to invest today in order to have \$25,000 in ten years if Joe's bank offers a 10-year CD that pays 1.8% interest compounded monthly.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$25,000 = P\left(1+\frac{0.018}{12}\right)^{12(10)}$$
$$25,000 = P(1.0015)^{120}$$
$$P \approx \$20,885$$

26. Bill and Sue plan to buy a home in 5 years. How much would they need to invest today at 1.2% compounded daily in order to have $30,000 in five years?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$30,000 = P\left(1+\frac{0.012}{365}\right)^{365(5)}$$
$$30,000 = P(1.000088)^{1825}$$
$$P \approx \$28,253$$

27. Suppose you invest $3000 in a four-year certificate of deposit (CD) that pays 1.5%interest compounded monthly the first 3 years and 2.2% compounded daily the last year. What is the value of the CD at the end of the four years?

$$A = P\left(1+\frac{r}{m}\right)^{mt}\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 3000\left(1+\frac{0.015}{12}\right)^{12(3)}\left(1+\frac{0.022}{365}\right)^{365(1)}$$
$$A = 3000(1.00125)^{36}(1.000060)^{365}$$
$$A \approx \$3208$$

28. Suppose you invest $8000 in a 10-year certificate of deposit (CD) that pays 1.25% interest compounded daily the first 6 years and 2% compounded continuously the last four years. What is the value of the CD at the end of the 10 years?

$$A = P\left(1+\frac{r}{m}\right)^{mt} e^{rt}$$
$$A = 8000\left(1+\frac{0.0125}{365}\right)^{365(6)} e^{(0.02)(4)}$$
$$A = 8000(1.000034)^{2190} e^{0.08}$$
$$A \approx \$9341$$

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Section 1-6 Logarithmic Functions

Goal: To solve problems that are logarithmic in nature

Properties of Logarithms

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \frac{x}{y}$

$\log_a x^n = n \log_a x$

Exponential Function: $f(x) = a^x,\ a > 0,\ a \neq 1$

$y = \log_a x$ means $x = a^y$

In Problems 1–20 find the value of *x*. (Evaluate to four decimal places if necessary.)

1. $\log_3 x = 4$

$$x = 3^4$$
$$x = 81$$

2. $\log_3(x+1) = 2$

$$x+1 = 3^2$$
$$x+1 = 9$$
$$x = 8$$

3. $\log_3 3^8 = 7 + 3x$

$$3^{7+3x} = 3^8$$
$$7 + 3x = 8$$
$$3x = 1$$
$$x = \frac{1}{3}$$

4. $\log_2 2^6 = 4 - 3x$

$$2^{4-3x} = 2^6$$
$$4 - 3x = 6$$
$$-3x = 2$$
$$x = -\frac{2}{3}$$

5. $\ln(x+6)=2$

$$e^2 = x+6$$
$$e^2-6=x$$
$$1.3891 \approx x$$

6. $\log_x(2x+3)=2$

$$x^2 = 2x+3$$
$$x^2-2x-3=0$$
$$(x-3)(x+1)=0$$
$$x=3$$
$$x \neq -1$$

The log function cannot have a negative base.

7. $\log_2(7x-19)=\log_2(2x+9)$

$$7x-19=2x+9$$
$$5x=28$$
$$x=5.6$$

8. $\log_3(8-5x)=\log_3(2x-13)$

$$8-5x=2x-13$$
$$21=7x$$
$$3=x$$

Since the value of 3 creates a negative log, there is no solution.

9. $\ln x+\ln 3=3$

$$\ln 3x=3$$
$$e^3=3x$$
$$\frac{e^3}{3}=x$$
$$6.6952 \approx x$$

10. $\ln x-\ln 3=1$

$$\ln \tfrac{x}{3}=1$$
$$e^1=\tfrac{x}{3}$$
$$3e=x$$
$$8.1548 \approx x$$

11. $\log(x-1)-\log 4=3$

$$\log \tfrac{x-1}{4}=3$$
$$10^3=\tfrac{x-1}{4}$$
$$(4)10^3=x-1$$
$$4(1000)+1=x$$
$$4001=x$$

12. $\ln(x-1)-\ln 6=2$

$$\ln \tfrac{x-1}{6}=2$$
$$e^2=\tfrac{x-1}{6}$$
$$6e^2=x-1$$
$$6e^2+1=x$$
$$45.3343 \approx x$$

13. $2^{3x} = 12$

$$\begin{aligned} \log 2^{3x} &= \log 12 \\ 3x \log 2 &= \log 12 \\ x &= \frac{\log 12}{3\log 2} \\ x &\approx 1.1950 \end{aligned}$$

14. $5^{x-1} = 17$

$$\begin{aligned} \log 5^{x-1} &= \log 17 \\ (x-1)\log 5 &= \log 17 \\ x\log 5 - \log 5 &= \log 17 \\ x\log 5 &= \log 17 + \log 5 \\ x &= \frac{\log 17 + \log 5}{\log 5} \\ x &\approx 2.7604 \end{aligned}$$

15. $7^{x-1} = 8^x$

$$\begin{aligned} \log 7^{x-1} &= \log 8^x \\ (x-1)\log 7 &= x\log 8 \\ x\log 7 - \log 7 &= x\log 8 \\ x\log 7 - x\log 8 &= \log 7 \\ x(\log 7 - \log 8) &= \log 7 \\ x &= \frac{\log 7}{\log 7 - \log 8} \\ x &\approx -14.5727 \end{aligned}$$

16. $4^{2x+3} = 5^{x-2}$

$$\begin{aligned} \log 4^{2x+3} &= \log 5^{x-2} \\ (2x+3)\log 4 &= (x-2)\log 5 \\ 2x\log 4 + 3\log 4 &= x\log 5 - 2\log 5 \\ 2x\log 4 - x\log 5 &= -2\log 5 - 3\log 4 \\ x(2\log 4 - \log 5) &= -2\log 5 - 3\log 4 \\ x &= \frac{-2\log 5 - 3\log 4}{2\log 4 - \log 5} \\ x &\approx -6.3429 \end{aligned}$$

17. $7^{x+1} = 10^{2x}$

$$\begin{aligned} \log 7^{x+1} &= \log 10^{2x} \\ (x+1)\log 7 &= 2x\log 10 \\ x\log 7 + \log 7 &= 2x\log 10 \\ x\log 7 - 2x\log 10 &= -\log 7 \\ x(\log 7 - 2\log 10) &= -\log 7 \\ x &= \frac{-\log 7}{\log 7 - 2\log 10} \\ x &\approx 0.7317 \end{aligned}$$

18. $e^{x+4} = 14.654$

$$\begin{aligned} \ln e^{x+4} &= \ln(14.654) \\ x+4 &= \ln(14.654) \\ x &= \ln(14.654) - 4 \\ x &\approx -1.3153 \end{aligned}$$

19. $x + 5 = e^3$

$$\begin{aligned} x &= e^3 - 5 \\ x &\approx 15.0855 \end{aligned}$$

20. $e^{4x} = e^{20}$

$$\begin{aligned} 4x &= 20 \\ x &= 5 \end{aligned}$$

21. You want to accumulate $20,000 by your son's eighteenth birthday. How much do you need to invest on the day he is born in an account that will pay 1.4% interest compounded quarterly? (Round your answer to the nearest dollar.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$20,000 = P\left(1+\frac{0.014}{4}\right)^{4(18)}$$

$$20,000 = P(1.0035)^{72}$$

$$\$15,552 \approx P$$

22. Using the information in Problem 21, how much would you need to invest if you waited until he is 10 years old to start the fund?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$20,000 = P\left(1+\frac{0.014}{4}\right)^{4(8)}$$

$$20,000 = P(1.0035)^{32}$$

$$\$17,884 \approx P$$

23. A bond that sells for $1000 today can be redeemed for $1200 in 10 years. If interest is compounded quarterly, what is the annual interest rate for this investment? (Round your answer to two decimal places when written as a percentage.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$1200 = 1000\left(1+\frac{r}{4}\right)^{4(10)}$$

$$1.2 = \left(1+\frac{r}{4}\right)^{40}$$

$$1.2^{1/40} = 1+\frac{r}{4}$$

$$1.2^{1/40} - 1 = \frac{r}{4}$$

$$4(1.2^{1/40} - 1) = r$$

$$1.83\% \approx r$$

24. A bond that sells for \$18,000 today can be redeemed for \$20,000 in 6 years. If interest is compounded monthly, what is the annual interest rate for this investment? (Round your answer to two decimal places when written as a percentage.)

$$\begin{aligned}
A &= P\left(1+\frac{r}{m}\right)^{mt} \\
20,000 &= 18,000\left(1+\frac{r}{12}\right)^{12(6)} \\
1.111 &= \left(1+\frac{r}{12}\right)^{72} \\
1.111^{1/72} &= 1+\frac{r}{12} \\
1.111^{1/72}-1 &= \frac{r}{12} \\
12(1.111^{1/72}-1) &= r \\
1.76\% &\approx r
\end{aligned}$$

25. What is the minimum number of months required for an investment of \$10,000 to grow to at least \$20,000 (double in value) if the investment earns 1.8% annual interest rate compounded monthly? What would be the actual value of the investment after that many months?

$$\begin{aligned}
A &= P\left(1+\frac{r}{m}\right)^{mt} \\
20,000 &= 10,000\left(1+\frac{0.018}{12}\right)^{12t} \\
2 &= (1.0015)^{12t} \\
\ln 2 &= \ln(1.0015)^{12t} \\
\ln 2 &= 12t\ln(1.0015) \\
\frac{\ln 2}{12\ln 1.0015} &= t \\
38.5 &\approx t
\end{aligned}$$

It will take about $(38.5)(12) = 462$ months to double.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 10,000\left(1+\frac{0.018}{12}\right)^{12(38.5)}$$
$$A = 10,000(1.0015)^{462}$$
$$A = 10,000(1.998668)$$
$$A \approx \$19,987$$

26. What is the minimum number of months required for an investment of $10,000 to grow to at least $30,000 (triple in value) if the investment earns 1.8% annual interest rate compounded monthly? What would be the actual value of the investment after that many months?

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$30,000 = 10,000\left(1+\frac{0.018}{12}\right)^{12t}$$
$$3 = (1.0015)^{12t}$$
$$\ln 3 = \ln(1.0015)^{12t}$$
$$\ln 3 = 12t\ln(1.0015)$$
$$\frac{\ln 3}{12\ln 1.0015} = t$$
$$61.1 \approx t$$

It will take about $(61.1)(12) = 733$ months to triple.

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$
$$A = 10,000\left(1+\frac{0.018}{12}\right)^{12(61.1)}$$
$$A = 10,000(1.0015)^{733}$$
$$A = 10,000(3.000192)$$
$$A \approx \$30,002$$

27. Some years ago Ms. Martinez invested $7000 at 2% compounded quarterly. The account now contains $10,000. How long ago did she start the account? (Round your answer UP to the next year.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$10,000 = 7000\left(1+\frac{0.02}{4}\right)^{4t}$$

$$\tfrac{10}{7} = (1.005)^{4t}$$

$$\ln\tfrac{10}{7} = \ln(1.005)^{4t}$$

$$\ln\tfrac{10}{7} = 4t\ln(1.005)$$

$$\frac{\ln\frac{10}{7}}{4\ln 1.005} = t$$

$$17.88 \approx t$$

It took approximately 18 years to have a balance of $10,000.

28. Some years ago Mr. Tang invested $18,000 at 3% compounded monthly. The account now contains $24,000. How long ago did he start the account? (Round your answer UP to the next year.)

$$A = P\left(1+\frac{r}{m}\right)^{mt}$$

$$24,000 = 18,000\left(1+\frac{0.03}{12}\right)^{12t}$$

$$\tfrac{4}{3} = (1.0025)^{12t}$$

$$\ln\tfrac{4}{3} = \ln(1.0025)^{12t}$$

$$\ln\tfrac{4}{3} = 12t\ln(1.0025)$$

$$\frac{\ln\frac{4}{3}}{12\ln 1.0025} = t$$

$$9.60 \approx t$$

It took approximately 10 years to have a balance of $24,000.

29. In a certain country the number of people above the poverty level is currently 25 million and growing at a rate of 4% annually. Assuming that the population is growing continuously, the population, P (in millions), t years from now, is determined by the formula:

$$P = 25e^{0.04t}$$

In how many years will there be 30 million people above the poverty level? 40 million? (Round your answers to nearest tenth of a year.)

<u>30 million people</u>

$$\begin{aligned} P &= 25e^{0.04t} \\ 30 &= 25e^{0.04t} \\ 1.2 &= e^{0.04t} \\ \ln 1.2 &= \ln e^{0.04t} \\ \ln 1.2 &= 0.04t \\ \frac{\ln 1.2}{0.04} &= t \\ 4.6 &\approx t \end{aligned}$$

<u>40 million people</u>

$$\begin{aligned} P &= 25e^{0.04t} \\ 40 &= 25e^{0.04t} \\ 1.6 &= e^{0.04t} \\ \ln 1.6 &= \ln e^{0.04t} \\ \ln 1.6 &= 0.04t \\ \frac{\ln 1.6}{0.04} &= t \\ 11.8 &\approx t \end{aligned}$$

It will take approximately 4.6 years to reach 30 million people and 11.8 years to reach 40 million people.

30. The number of bacteria present in a culture at time t is given by the formula $N = 20e^{0.35t}$, where t is in hours. How many bacteria are present initially (that is when t = 0)? How many are present after 24 hours? How many hours does it take for the bacteria population to double? (Round your answers to nearest whole number.)

Initially there are $N = 20e^{0.35(0)} = 20e^{0} = 20$ bacteria present.
After 24 hours there will be $N = 20e^{0.35(24)} = 20e^{8.4} = 20(4447.066748) = 88{,}941$ bacteria present.

$$N = 20e^{0.35t}$$
$$40 = 20e^{0.35t}$$
$$2 = e^{0.35t}$$
$$\ln 2 = \ln e^{0.35t}$$
$$\ln 2 = 0.35t$$
$$\frac{\ln 2}{0.35} = t$$
$$1.98 \approx t$$

The number of bacteria will double after approximately 2 hours.